PROPAGATION OF A TWO-DIMENSIONAL SUPERSONIC RADIATION WAVE

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§1. For sufficiently high flux densities in the radiation of an optical quantum generator (OQG) the hydrodynamic mechanism of the propagation of the plasma boundary toward the ray (the luminescence detonation) will be replaced by a radiation mechanism. The plasma, which has been heated to a high temperature, emits intense ultraviolet radiation. Such radiation is absorbed in the cold layers of the air surrounding the plasma, so that the air layers are heated and ionized and absorb the laser radiation. As a result, these layers themselves begin to emit high-temperature radiation and so on – a radiation wave is propagated through the gas in a direction opposite to that of the beam. If the velocity of propagation of the plasma front is much higher than the velocity of sound in the hot plasma (a fast supersonic wave), the hydrodynamic processes will not produce any substantial effect on the parameters of the heated gas, the density of which will be close to the original density in the undisturbed state.

Raizer [1, 2] gives approximate estimates for the parameters of a supersonic radiation wave for the case in which the beam diameter and the transverse dimension of the plasma cloud are small and the cloud is greatly elongated along the beam. He assumed that the hot plasma emits a volumetric radiation and that the radiation reaching the plasma front is essentially radiation from distances of the order of the beam diameter.

In [3], calculations of the radiation-gas dynamics problem were carried out for the case in which the beam diameter is fairly large and the situation is close to a plane situation. In the plane case, for a sufficiently long time of action, the optical thickness of the plasma layer approaches unity, and therefore the maximum fluxes of the radiation generated in the hot plasma are close to the fluxes of black-body radiation at a corresponding temperature and much larger than the fluxes of radiation from an optically thin volume; the velocity of propagation of the wave will also be correspondingly greater.

Conditions close to plane conditions will necessarily exist if the beam radius R_0 is much larger than the traversed distance L. However, the following question arises: Is the condition $R_0 \gg L$ a necessary one?

It is therefore desirable to investigate in detail the effect of two-dimensionality on the propagation of the plasma flare for finite beam diameter.

When the density of the medium remains unchanged, for the calculation of the problem concerning the propagation of a two-dimensional radiation wave the propagation equations of the radiation must be solved simultaneously with the energy equation alone, not with a complete system of hydrodynamic equations.

The energy equation for $\rho = \rho_0$=const has the form

$$\rho\partial e/\partial t = -\operatorname{div}\mathbf{F}, \tag{1.1}$$

where e is the specific internal energy (of a unit mass); **F** is the flux density of the radiant energy. The radiation propagation equation for the spectral density of the radiation I_ν, propagated in the direction Ω, has the form

$$\partial I_\nu/\partial\Omega = -k_\nu(I_\nu - B_\nu), \tag{1.2}$$

where k_ν is the linear coefficient of absorption; $B_\nu(\nu, T)$ is the Planck function. The connection between the flux density and the intensity I_ν is given by the expression

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$$\mathbf{F} = \int d\boldsymbol{\Omega} \int_0^\infty I_\nu \boldsymbol{\Omega} d\nu. \tag{1.3}$$

The system (1.1)-(1.3) is supplemented by some information concerning the optical and thermodynamic properties of the medium, i.e., by the equation of state $T = T(e, \rho)$ and the functional relations giving the absorption coefficients $k_\nu = k_\nu(\nu, T, \rho)$. In order to solve the problem, we used a previously developed method for solving problems concerning the heating and cooling of a fixed gaseous medium (or one moving according to a specified law) by a nonequilibrium radiation in the two-dimensional axially symmetric case. This method is a generalization of a similar method for the spherically symmetric problem described in [4]. Since it can also be used in solving other two-dimensional problems relating to the propagation of fast thermal waves with nonequilibrium radiation or can be a component part of a method for the solution of analogous problems with a complete system of gasdynamic equations, we shall describe it briefly.

§2. A cylindrical volume is subdivided into a countable number of cells by planes perpendicular to the axis of symmetry (z axis) and divided into annuli by cylindrical "tubes" along the axis. Each such cell with indices s, j is defined by specifying the two planes perpendicular to the z axis which have the coordinates z_j and z_{j+1} (where j is the number of the disk, counted from the base of the cylinder) and specifying the inner radius r_s and the outer radius r_{s+1} of the tube (where s is the number of tube).

We assume that within each cell the energy, density, and absorption coefficients are constant, the temperature varies according to a parabolic law, and the Planck function has the form

$$B = a + br^2 + cz^2, \tag{2.1}$$

where the constants a, b, and c are determined by the Planck functions corresponding to the temperatures at the boundaries of the cell. For simplicity of notation, here and hereafter we shall omit the frequency index.

Let us consider the solution of the radiation propagation equation (1.2). For the intensity of the radiation at the point Ω in the direction $\boldsymbol{\Omega}$ the equation has the form

$$I(\Omega) = I(\Omega_0)\, e^{-\int_{\Omega_0}^{\Omega} k(\Omega')d\Omega'} + \int_{\Omega_0}^{\Omega} k(\Omega')\, B(\Omega')\, e^{-\int_{\Omega'}^{\Omega} k(\Omega'')d\Omega''}\, d\Omega'. \tag{2.2}$$

Substituting into (2.2) the Planck function in the form (2.1) and integrating over an interval of the beam $\boldsymbol{\Omega}$ from the point 1 with coordinates r_1 and z_1 to the point 2 with coordinates r_2 and z_2, we obtain an expression for the intensity of the radiation at the point 2:

$$I_2 = I_1 e^{-kD} + B_2 - B_1 e^{-kD} + \frac{1}{kD}\Big\{b\,(r_1^2 - r_2^2) + \frac{2}{kD}\,[bd^2 +$$

$$+ c\,(z_2 - z_1)^2]\Big\}\left(1 - e^{-kD}\right) - \frac{bd^2}{kD}\left(1 + e^{-kD}\right) - \frac{2c}{kD}\,(z_2 - z_1)\left(z_2 - z_1 e^{-kD}\right), \tag{2.3}$$

where I_1, I_2, B_1, B_2 are the values of the radiation intensity and the Planck function at the points 1 and 2; D is the length of the interval of the beam $\boldsymbol{\Omega}$ between the points 1 and 2; and d is the length of the projection of this beam onto a plane perpendicular to the z axis.

We introduce the angle $0 \le \varphi \le \pi$, formed by the direction of the radiation Ω with the z axis, and the angle $0 \le \theta \le \pi$, formed by the projection of the beam $\boldsymbol{\Omega}$ onto the plane perpendicular to the z axis with the radius-vector $\mathbf{r}$ in this plane.

In the case of axial symmetry the radiation field is independent of the polar angle, and consequently the angles φ and θ completely determine the direction of the radiation at each point of space with coordinates r and z.

We express the constants and functions in the expression (2.3) in terms of the coordinates r and z and the angles φ and θ for the case in which the point 2 coincides with the point s, j (i.e., $z_2 = z_j$ and $r_2 = r_s$), and the direction of the radiation $\boldsymbol{\Omega}$ is given by the angles θ and φ at this point. The point 1 is on the boundary surface of the cell under consideration and the values of its coordinates depend on the angles θ and φ. Thus,

$$r_{s-1} \leqslant r_1 \leqslant r_s \text{ for } 0 \leqslant \theta \leqslant \pi/2 \text{ and } r_s \leqslant r_1 \leqslant r_{s+1} \text{ for } \pi/2 \leqslant \theta \leqslant \pi;$$

$$z_{j-1} \leqslant z_1 \leqslant z_j \text{ for } 0 \leqslant \varphi \leqslant \pi/2 \text{ and } z_j \leqslant z_1 \leqslant z_{j+1} \text{ for } \pi/2 \leqslant \varphi \leqslant \pi.$$

TABLE 1

	$\lvert\cos\varphi\rvert \geqslant \lvert\cos\varphi_k\rvert$	$\lvert\cos\varphi\rvert < \lvert\cos\varphi_k\rvert$
D	$(z_j - z_i)/\cos\varphi$	$\delta/\lvert\sin\varphi\rvert$
d	$D\lvert\sin\varphi\rvert$	δ
z_1	z_i	$z_j - D\cos\varphi$
r_1	$r_s^2 + d^2 - 2dr_s\cos\theta$	r_p^2
b	$(B_{s,i} - B_{p,i})/(r_s^2 - r_p^2)$	$(B_{s,j} - B_{p,j})/(r_s^2 - r_p^2)$
c	$(B_{s,j} - B_{s,i})/(z_j^2 - z_i^2)$	$(B_{p,j} - B_{p,i})/(z_j^2 - z_i^2)$
B_1	$B_{s,i} - b(r_s^2 - r_1^2)$	$B_{p,j} - c(z_j^2 - z_1^2)$
I_1	$I_{s,i}(\theta,\varphi) + (I_{p,i}(\alpha,\varphi) - I_{s,i}(\theta,\varphi))\frac{\alpha}{\delta}$	$I_{p,j}(\alpha,\varphi) + (I_{p,i}(\alpha,\varphi) - I_{p,j}(\alpha,\varphi))\frac{z_j - z_1}{z_j - z_i}$.

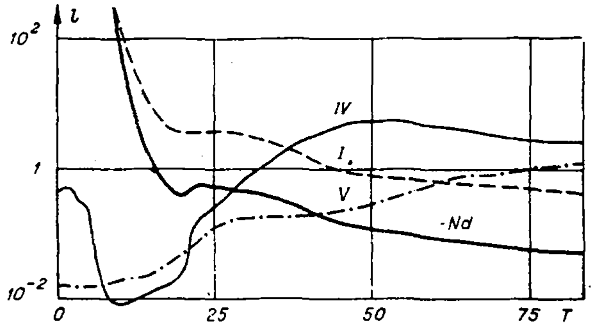

Fig. 1

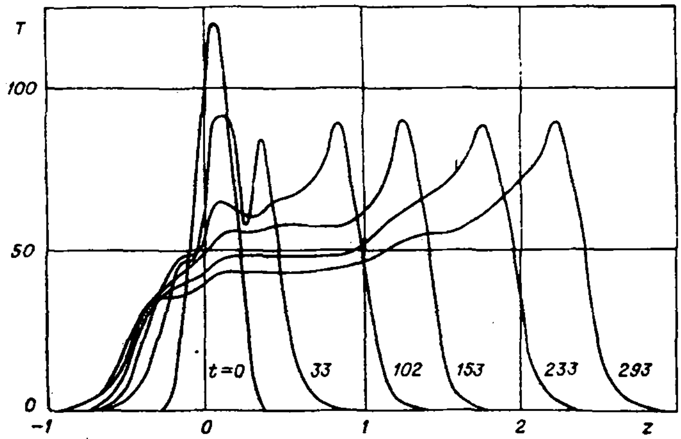

Fig. 2

The connection between the geometric quantities in the plane perpendicular to the z axis is the same as in the spherically symmetric case [4]:

$$\lvert\cos\alpha\rvert = \sqrt{1 - (r_s/r_p)^2\sin^2\theta}, \qquad p = \begin{cases} s-1 & 0 \leqslant \theta \leqslant \theta_2 \\ s \quad \text{for} & \theta_2 \leqslant \theta \leqslant \pi/2 \\ s+1 & \pi/2 \leqslant \theta \leqslant \pi, \end{cases}$$

$$\delta = r_s\cos\theta - r_p\cos\alpha,$$

where α is the angle formed by the projection of the beam Ω with the radius-vector at the point r_p; δ is the dimension of the cell along the projection of the beam Ω; $\theta_2 = \arcsin(r_{s-1}/r_s)$.

Depending on the value of the angle φ, the constants and functions in the expression (2.3) take on the values shown in Table 1, where the values of the angle φ_k are determined from the relation

$$\cos\varphi_k = \frac{z_j - z_i}{\sqrt{\delta^2 + (z_j - z_i)^2}}, \text{ where } i = \begin{cases} j-1 & \varphi < \pi/2 \\ & \text{for} \\ j+1 & \varphi > \pi/2. \end{cases}$$

By substituting into (2.3) the values of the functions shown in Table 1, we can obtain an expression for determining the radiation field in the entire region under consideration.

The calculation of the propagation equation for a given intensity of the radiation hitting the boundary of the region is carried out from this boundary to the center of the region. Thus, for each angle $\pi/2 \le \theta \le \pi$ we determine the radiation intensities in the angular interval $0 \le \varphi \le \pi/2$ for all annuli of a particular cylindrical tube in succession from the bottom annuli to the top ones, and then for the same annuli in the opposite direction in the angular interval $\pi/2 \le \varphi \le \pi$.

The resulting intensity values for a cylindrical surface with radius r_s in the angular interval $\pi/2 \le \theta \le \pi$ are the initial values not only for the points with radius $r_{s-1} < r_s$ in the same interval of θ values, but also for all points with radius $r_l \ge r_s$ in the angular interval $\theta_2 \le \theta \le \theta_1$, where

$$\cos\theta_1 = \sqrt{1 - (r_s/r_l)^2};\quad \cos\theta_2 = \sqrt{1 - (r_{s-1}/r_l)^2}.$$

The proposed scheme for calculating the propagation equation enables us to reduce sharply the size of the machine memory required for an accurate calculation of the radiation field, since with this approach a set of spectral intensities which is complete for the angular values is stored at each point z_j for only one radius, while for all the other points with different radii we store values which are integral with respect to the angles – the radiation fluxes.

The radiation flux densities integrated over the entire frequency spectrum are substituted into the right side of the energy equation (1.1).

To solve the energy equation, we used Euler's method with conversion.

§3. Using the above-described method, we carried out the calculation of the problem of the propagation of a supersonic radiation wave in air with density equal to 0.1 times the normal density, brought about by the action of radiation from a neodymium OQG ($h\nu$ = 1.16 eV) with a constant flux density of q_0 = 1000 MW/cm^2 and a pulse length of about 300 nsec, i.e., for the same conditions as we used in [3] but for a finite beam radius R_0 = 0.4 cm.

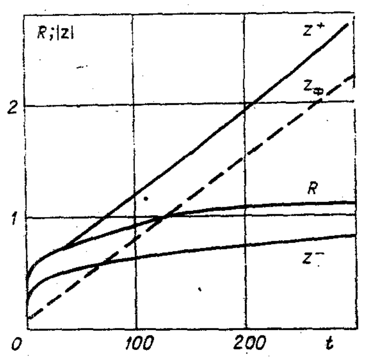

Fig. 3

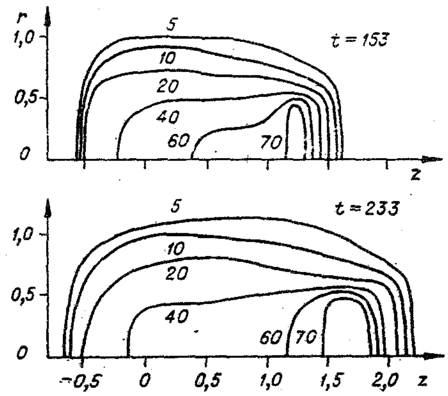

Fig. 4

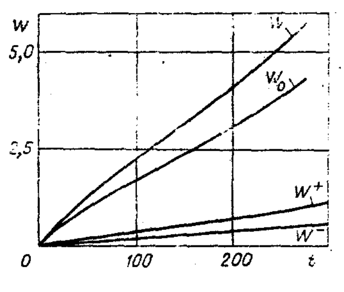

Fig. 5

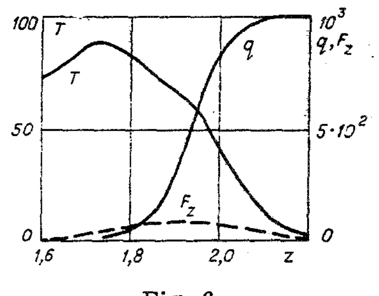

Fig. 6

As the equation of state of the air, we used the table from [5]. The radiation propagation equation was solved for six groups (the same as in [3]), which were bounded by the following values of quantum energy: 0 ... 6.52 ... 7.95 ... 9.96 ... 18.6 ... 80.5 ... 248eV. The absorption coefficients for these groups were obtained by averaging by Planck's method the tables of the optical properties of air [6].

The functions showing the variation of the radiation path length l as a function of temperature T for spectral groups I, IV, and V, as well as for the radiation of the neodymium laser, are shown in Fig. 1.

The propagation of the radiation in hot plasma with a temperature of about 5-10 eV takes place essentially in groups IV and V; the radiation of these groups is strongly absorbed in the cold layers of air, which ensures passage through the plasma boundary in a direction opposite to the laser radiation. For temperatures higher than 3-4 eV the path length of the laser radiation is less than the path length of the radiation in these groups, and the heating of the plasma at these and higher temperatures is due to the absorption of the laser radiation energy. The radiation path length in the first group, when the temperature decreases, will increase in approximately the same way as the laser radiation path length. Low-temperature plasma and cold air are transparent to the quanta of this group. The radiation belonging to this group ensures the loss of energy from the plasma flare.

As can be seen from Fig. 1, for high temperatures the average Planck path lengths of the radiation in all of the above-mentioned groups are of the order of 0.5-2 cm, i.e., plasma with the same characteristic dimensions is semitransparent to the radiation. In order to determine the radiation energy fluxes in the given cases, we must solve the radiation propagation equation, since we cannot use either a radiative-heat-conduction approximation or a volumetric-luminescence approximation. This is all the more true for the plasma-front region, where the radiation is essentially of the nonequilibrium type.

The number of spectral groups was selected with a view to ensuring comparability with the results of the plane problem [3] and keeping the problem within the limitations of the BÉSM-4 computer that was used.

At the initial instant of time the plasma layer was considered to be cylindrical with a radius equal to the radius of the beam, i.e., 0.4 cm, uniformly heated along the radius. The temperature distribution along the z-coordinate was taken from the solution of the one-dimensional plane problem [3] for an analogous variant at a time equal to about 50 nsec. This instant of time was taken to be the zero time value for the calculation. The original thickness of the plasma layer was L = 0.5 cm, i.e., only slightly more than half the diameter of the cylinder and the beam. The initial energy was about 25 J; at the end of the action of the laser pulse (300 nsec) the energy was about 175 J. The laser radiation was propagated along the z axis in the negative direction.

The total number of calculation layers with respect to time was more than 50. The distance step (both along the radius and along the z axis) was 0.1 cm. The propagation equation was integrated along 50 beams in both directions (10 values of the angle φ from 0 to π and five values of the angle θ from 0 to $\pi/2$) for each of the six spectral groups.

The calculation results are shown in Figs. 2-6, where t is the time in nsec; r is the radius and z is the axial coordinate in cm; T is the temperature in 10^3 °K; W is the amount of energy lost in the radiation in J; and q, F are the laser and characteristics flux densities of the radiation in MW/cm^2.

Figure 2 shows the distributions of the temperature along the axis of symmetry (with respect to z for r = 0) for different instants of time. The picture is qualitatively close to the one obtained earlier in the solution of the one-dimensional problem [3]: We arrive fairly rapidly at a quasistationary regime of propagation of the plasma front in a direction opposite to the radiation, and the maximum temperature remains practically unchanged. The velocity of its propagation (~ 70 km/sec) and the maximum temperature behind the front (90,000 °K) are close to the corresponding values for the plane case.

Figure 3 shows the variation with time of the boundary of the plasma region (more precisely, of points with a temperature of 3000°K) in three characteristic directions – toward the laser beam (z^+), in the opposite direction (z^-), and in the direction of R, perpendicular to the axis of symmetry (z = 0). By time t = 300 nsec the length of the plasma flare ($z^+ + |z^-|$) amounted to 3.5 cm, which is more than four times the diameter of the irradiated spot; in the lateral direction the dimension of the flare increased to almost three times the original dimension. The dashed-dot curve corresponds to the advance of the zone with maximum temperature behind the front.

Figure 4 shows the isotherms characteristic of the spatial distribution of the temperature within the flare for two instants of time – 153 and 233 nsec.

The energy losses from the flare as a result of the radiation are shown in Fig. 5 as functions of time. The value W of the emitted energy was found from the integral fluxes emerging through the end faces and the lateral surface of the cylinder within which the flare was situated. The value of the energy exiting through the lateral surface is indicated by subscript "0", the energy through the end face toward the laser beam by the superscript "+" and the energy in the opposite direction by the superscript "−". The main energy loss takes place as a result of the emission through the lateral surface. However, the total value of energy lost, $W = W_0 + W^+ + W^-$, was found to be fairly small. Thus, by 300 nsec we have $W \simeq 7$ J, which is only about 4% of the energy supplied to the OQG.

Figure 6 shows the distribution along z (for $r = 0$) of the flux density q of the laser radiation and the flux density F_z of the characteristic radiation, as well as the temperature T near the advancing plasma front for time $t = 233$ nsec. It can be seen that the laser radiation begins to be markedly absorbed in the region with temperature higher than 10,000-15,000°K. It follows from Fig. 6 that the characteristic depth of penetration of the laser radiation into the plasma (when q decreases by a factor of e) is ~ 0.2 cm. About the same value is found for the distance from the point where the flux of characteristic radiation reached its maximum to the point where it is practically completely absorbed. The temperature reaches its maximum value at distances of ~ 0.4 cm from the point where the intensive absorption of the laser radiation begins.

Thus, the characteristic width of the plasma front Δ is 0.2-0.4 cm, i.e., less than the radius of the beam. Moreover, the fluxes of characteristic radiation are in general much lower than q_0; they are only enough to ensure heating of the plasma to the "ignition" temperature [1-3] and initiate the absorption of the laser, while essentially the front moves because of the laser energy. This is also the explanation for the elongated nature of the flare – expansion to the side takes place only as a result of the fluxes F_r, which are on the order of F_z or less. It becomes clear why in this case the advancing plasma front moves with a velocity close to the velocity in the plane case: The effects of two-dimensionality manifest themselves only at a great distance from the advancing front, outside the region in which the laser radiation energy is produced; therefore, the criterion of closeness to the plane case is the condition $\Delta \lessdot R_0$, not $L \lessdot R_0$.

Since the motion of the flare boundaries in the lateral direction and backward is gradually slowed down, the accumulating hydrodynamic disturbances lead, in the final analysis, to the formation of a shock wave and an expansion of the plasma by a hydrodynamic mechanism. However, for $\Delta \lessdot R_0$ this process has no effect on the motion of the advancing front by the radiation mechanism.

Thus, for energies in the order of 100-1000 J we can observe the motion of plasma fronts by the radiation mechanism at velocities substantially greater than the velocity of propagation of luminescence-detonation waves. Such energies are within the limits that have already been attained today (see, for example, [7]). It would be of interest to conduct appropriate experiments and to compare their results with the theoretical predictions given above.

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